

# Multicast Routing for Decentralized Control of Cyber Physical Systems with an Application in Smart Grid

Husheng Li, Lifeng Lai, and H. Vincent Poor

**Abstract**—In cyber physical systems, communication is needed for conveying sensor observations to controllers; thus, the design of the communication sub-system is of key importance for the stabilization of system dynamics. In this paper, multicast routing is studied for networking of decentralized sensors and controllers. The challenges of uncertain destinations and multiple routing modes, which are significantly different from traditional data networks, are addressed by employing the theories of hybrid systems and linear matrix inequalities, thus forming a novel framework for studying the communication sub-system in cyber physical systems. Both cases of negligible delay and non-negligible delay are discussed. The proposed framework is then applied in the context of voltage control in smart grid. Numerical simulations using a 4-bus power grid model show that the proposed framework and algorithm can effectively stabilize cyber physical systems.

**Index Terms**—Smart grid, voltage control and routing.

## I. INTRODUCTION

IN RECENT years, cyber physical systems (also known as networked control systems in many scenarios), which consist of computing and physical sub-systems, have received considerable attention [15] due to their wide applications in various areas such as power grids [11], robotic networks [4] and embedded systems [23]. A typical cyber physical system has the capabilities of sensing, controlling and communication. In many cases, the sensor(s) and controller(s) are not located at the same place. Hence, a communication network is needed to convey the system observations from the sensor(s) to the controller(s).

An important application of cyber physical systems is smart grid [11], in which the power grid is the physical world while the communication system for monitoring and controlling the power grid bridges the physical world and computing systems. An example in the context of voltage control in smart grid is shown in Fig. 1, where there are three sensors, three relay nodes and three distributed energy

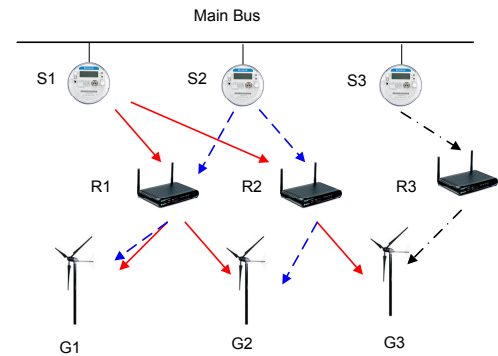


Fig. 1. An illustration of communications in cyber physical systems: a case study of smart grid. Different data flows are represented by different arrows.

generators (DEGs) which play the role of controllers. The communication network conveys the sensor observations, i.e., the voltage measurements at the points of common coupling (PCCs) at the main transmission line, to the controllers in order to stabilize the voltage to a reference value.

As in traditional data communication networks or sensor networks, routing is needed to find paths from the sensors to the controllers. In particular, an observation at a sensor may need to be sent to multiple controllers. For example, two or more DEGs may need to monitor the voltage at one point in order to take control actions (increasing or decreasing their voltages) since their control actions are coupled in the voltage dynamics. Hence, *multicast* routing [5] [12] [24] is needed<sup>1</sup>. In a sharp contrast to traditional communication networks, multicast routing in cyber physical systems is challenging due to the following issues:

- **Uncertain destinations:** In traditional networks, the destinations are known in advance; e.g., in video streaming, the destinations are the customers requesting the video clip. However, in cyber physical systems, the destinations are unknown in advance, which consists of one of the most important design parameters. If the bandwidth allows, it is desirable to broadcast the observations of each sensor to all controllers since the control actions may be coupled and a joint control may better stabilize the system. When the bandwidth is limited, *it is challenging to determine the set of destinations for each sensor*. For example, in Fig. 1, DEG G3 has weak impacts on the voltages at S1 and S2 but a strong impact on the voltage

<sup>1</sup>When there are sufficient communication resources, it is desirable for each sensor to broadcast its observations to all controllers. However, in many cases there are insufficient resources or there are many sensors and controllers (e.g., there could be tens of DEGs in a microgrid); then we can only do multicast.

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H. Li is with the Department of Electrical Engineering and Computer Science, the University of Tennessee, Knoxville, TN, 37996 (e-mail: husheng@eecs.utk.edu).

L. Lai is with the Department of Systems Engineering, University of Arkansas, Little Rock, AR 72204 USA (Email: lxlai@ualr.edu).

H. V. Poor is with the Department of Electrical Engineering, Princeton University, Princeton, NJ, 08544 (Email: poor@princeton.edu).

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at S3. Thus, if the bandwidth is sufficiently large, we can build routes from S1, S2 and S3 to G3, which can improve the performance of control; otherwise, we need to consider only the path from S3 to G3, at the cost of some performance loss.

- Multiple routing modes<sup>2</sup>: In traditional networks, usually the multicast routing process results in one set of paths, which we call a routing mode, for the sources and destinations. However, in cyber physical systems, *a single routing mode may not stabilize the cyber physical system*. It is possible that we choose multiple routing modes and let the communication network switch its operation among these routing modes in an adaptive manner. From the viewpoint of system theory, each routing mode corresponds to a mode of the system dynamics. According to the theory of hybrid systems [20], switching among multiple unstable system modes may result in stable dynamics. A trivial solution is to consider all possible routing modes and find the switching rules among all these routing modes; however, the set of modes will be prohibitively large for a large network. Hence, the challenge is *how to find a reasonable set of routing modes for stable system dynamics*.

In this paper, we *study multicast routing in cyber physical systems, concentrating particularly on the above two challenges, namely the determination of destinations and routing modes*. We will formulate routing as an optimization problem, by employing the theories of hybrid systems and linear matrix inequalities (LMIs), and then solve the optimization problem using heuristic approaches. Although there have been some studies on the design of communication for networked control systems [18] [19] [26]<sup>3</sup>, they are mostly focused on the physical layer and consider only centralized controls. In [8], the routing problem is studied for the purpose of system state estimation. Unlike our study, [8] considers a single and fixed destination controller; moreover, it assumes that there is only one sensor such that the conflict among multiple data flows can be ignored. In [14], the communication topology is designed for distributed control, where the communication delay is ignored. To our knowledge, this paper is the first to study networking in generic cyber physical systems with decentralized sensors and decentralized controllers<sup>4</sup>. The proposed framework shrinks the gap between the communities of communication and control and sheds light for future studies of communication system design for cyber physical systems. Based on the study of generic cyber physical systems, we will apply the proposed framework in the context of voltage control [1] [17] for DEGs in smart grid, as a case study. The proposed framework can also be applied in other controls of power grid, e.g., frequency control [27], since they are all special cases of controlling cyber physical systems.

The remainder of this paper is organized as follows. The

model for cyber physical systems will be explained in Section II. The routing algorithm with ignorable delay will be discussed in Section III. Then, the study will be extended to the more generic case in which the delays are non-negligible. Numerical simulations will be carried out in the context of voltage control in smart grid in Section VI. Conclusions will be drawn in Section VII.

## II. SYSTEM MODEL

In this section, we introduce the system model which contains the physical dynamics and the communication network. Certain assumptions will be made to simplify the analysis without losing the essence of the study.

### A. Model of Physical Dynamics

We assume that there are  $N_c$  controllers and  $N_s$  sensors in the cyber physical system, all being decentralized. For simplicity, we assume that the control action at each controller is scalar, as is the observation at each sensor. This simplifies the mathematical notation and can be straightforwardly extended to the generic case of vector control actions or vector observations.

For simplicity, we assume that the dynamics of the physical sub-system are linear and free of perturbations, and are given by

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \\ \mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) \end{cases}, \quad (1)$$

where  $\mathbf{x}$  is an  $M$ -vector representing the system state;  $\mathbf{u}$  is the  $N_c$ -dimensional vector of control actions where  $u_{n_c}$  stands for the control action of controller  $n_c$ ;  $\mathbf{y}$  is the  $N_s$ -dimensional vector of observations at the sensors where  $y_{n_s}$  is the observation at sensor  $n_s$ . The dimensions of matrices  $\mathbf{A}$ ,  $\mathbf{B}$  and  $\mathbf{C}$  are  $M \times M$ ,  $M \times N_c$  and  $N_s \times M$ , respectively. Note that the assumption of linear dynamics is valid for linear systems and is also effective for nonlinear systems when the system state deviates only slightly from an equilibrium point. It is much more challenging to study the general nonlinear case, which is a topic for future work. We also note that we do not consider random perturbations, such as noise in the observation, in the system. The only uncertainty is the initial condition in the system dynamics (otherwise, if the initial state is also known, there is no need for communications). The study of this deterministic system will be extended to stochastic systems in future.

### B. Model of Communication Network

We assume that the  $N_c$  controllers and  $N_s$  sensors are all equipped with communication interfaces, either wired or wireless. There are also  $N_r$  relay nodes in the communication network, which can help the delivery of observations from the sensors to the controllers. We denote the three types of nodes by  $\{n_c\}_{n_c=1,\dots,N_c}$ ,  $\{n_s\}_{n_s=1,\dots,N_s}$ , and  $\{n_r\}_{n_r=1,\dots,N_r}$ , where the subscripts represent the types of nodes. We call the data flow from a sensor to a controller a *connection*. The topology of the communication network composed of the three types of nodes is known in advance. For the wired network, the topology is determined by the

<sup>2</sup>One routing mode means one distinct selection of paths for the source nodes to destination nodes.

<sup>3</sup>Given a communication network, the impact of communication imperfections like delay and packet drop [9] has been intensively studied.

<sup>4</sup>Note that the decentralization in this paper is for the sensing and control. The routing scheme is still centralized. Decentralized routing is much more complicated, and we plan to study it in the future.

existence of wired links; for the wireless network, the topology is determined by the distances among the nodes. We use the notation  $a \sim b$  to denote that nodes  $a$  and  $b$  are directly connected in the communication network.

We make the following assumptions on the communication network throughout the paper in order to simplify the analysis:

- **Fluid Traffic:** We consider the data flows from sensors to controllers to be continuous; i.e., we ignore the details of sampling, quantization and possible packet dropout. Although the impacts of sampling interval, quantization error and packet drop probability on the system dynamics have been intensively studied in the area of networked control [9], it renders the analysis prohibitively complicated. This fluid traffic assumption can simplify the analysis and is valid when the sampling rate is high, the quantization error is very small, and the communication channels are of very good quality.
- **Bandwidth Constraint:** We assume that transmitting the data flow of one sensor requires one unit of bandwidth. The bandwidth of the communication link between nodes  $a$  and  $b$  is assumed to be an integer and is denoted by  $w_{ab}$  (in units of bandwidth). Hence, the link can support data flows from at most  $w_{ab}$  sensors, namely

$$\sum_{n_s=1}^{N_s} I(n_s, a, b) \leq w_{ab}, \quad (2)$$

where  $I(n_s, a, b)$  equals 1 if the data flow of sensor  $n_s$  passes through link  $ab$ ; otherwise,  $I(n_s, a, b) = 0$ .

- **Routing Mode Switching:** We assume that there are totally  $Q$  different routing modes, and that the routing mode can be switched every  $\tau$  seconds. For simplicity, we consider a simple round-robin switching policy; i.e., the routing mode is selected in the order 1, 2, ...,  $Q$ . The performance can be improved by adaptively selecting the routing mode. However, the mode decision needs to be carried out in a decentralized manner, which is beyond the scope of this paper<sup>5</sup>.

### III. MULTICAST ROUTING WITHOUT DELAY

In this section, we study multicast routing under the assumption that there is no delay and the observation can be delivered to the controller instantaneously without any loss. The assumption of no delay simplifies the analysis, which provides insights for the more complicated case with non-negligible delay. This assumption is also valid when the communication speed is very fast, compared with the dynamics of the physical system. For example, the electromechanical dynamics could be on the order of seconds (Page 6, [21]), while the wireless network could deliver the data in milliseconds. The case of non-negligible delay will be studied in the next section.

We will discuss both cases of single routing mode and multiple routing modes. We will first discuss decentralized control, thus obtaining the system dynamics that are determined by the routing mode. Then, we will formulate multicast routing as a series of optimization problems. Finally, we propose heuristic algorithms for solving the optimization problems.

<sup>5</sup>In [16], we have studied distributed scheduling for a single hop network. The extension to the multihop case is highly nontrivial.

#### A. Decentralized Control

Here, we introduce the mechanism of decentralized control in the cyber physical system.

1) *Single Routing Mode Case:* We first consider the single routing mode. We assume that a linear feedback control is employed for the cyber physical system, which is given by [28]

$$\mathbf{u}(t) = \mathbf{K}\mathbf{y}(t), \quad (3)$$

where  $\mathbf{K}$  is a constant feedback gain matrix. Note that  $\mathbf{K}$  is dependent on the routing mode. Since we consider a decentralized control, the matrix  $\mathbf{K}$  has a special structure, i.e.,

$$\mathbf{K}_{ij} = 0, \quad (4)$$

if there is no connection between sensor  $j$  and controller  $i$ , such that the control action  $u_i$  is independent of the observation  $y_j$ . Equivalently, the nonzero elements of the  $i$ -th row of  $\mathbf{K}$  correspond to the sensors having connections to controller  $i$ . For instance, in the example of Fig. 1, suppose that the data from sensor 1 is sent to controllers 1 and 2 while sensors 2 and 3 are connected to controllers 2 and 3, respectively. Then, the matrix  $\mathbf{K}$  is given by

$$\mathbf{K} = \begin{pmatrix} K_{11} & K_{12} & 0 \\ 0 & K_{22} & 0 \\ 0 & 0 & K_{33} \end{pmatrix}. \quad (5)$$

Substituting (3) into (1), we obtain the system dynamics which are given by

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{K}\mathbf{C}\mathbf{x}(t) \\ &= \tilde{\mathbf{A}}\mathbf{x}(t), \end{aligned} \quad (6)$$

where  $\tilde{\mathbf{A}} \triangleq \mathbf{A} + \mathbf{B}\mathbf{K}\mathbf{C}$ .

2) *Multiple Routing Modes:* Recall that there are totally  $Q$  routing modes. We denote by  $\mathbf{K}_q$  the feedback gain matrix corresponding to the  $q$ -th routing mode. Then, the system dynamics can be written as

$$\dot{\mathbf{x}}(t) = \tilde{\mathbf{A}}_{q(t)}\mathbf{x}(t), \quad (7)$$

where  $\tilde{\mathbf{A}}_{q(t)} \triangleq \mathbf{A} + \mathbf{B}\mathbf{K}_{q(t)}\mathbf{C}$  and  $q(t)$  is the routing mode at time  $t$ . Obviously, (7) represents the dynamics of a hybrid system with  $Q$  modes in which  $\mathbf{x}(t)$  is the continuous system state and  $q(t)$  is the discrete system state.

#### B. Optimization Problem Formulation

Here, we study how to select the routing mode(s) in order to stabilize the cyber physical system using the decentralized feedback control.

1) *Single Routing Mode:* We first consider the case in which a single routing mode can stabilize the system. The key challenge is how to determine the set of destinations. Given an arbitrary routing mode, if the feedback gain matrix  $\mathbf{K}$  is determined in advance<sup>6</sup>, we can simply check the eigenvalues of  $\tilde{\mathbf{A}}$ : if the real parts of all eigenvalues are in the left half

<sup>6</sup>For example, we can first determine the matrix  $\mathbf{K}_0$  when all observations can be broadcast to all controllers such that all elements in  $\mathbf{K}_0$  are free variables, e.g., computing  $\mathbf{K}_0$  using the LQR control. Then, we obtain  $\mathbf{K}$  for the given routing mode by eliminating the elements from unobserved sensors.

of complex plane, the system is stable (here stability means that all trajectories of the dynamics converge to a unique equilibrium point [3]); otherwise, it is unstable. In this paper, we consider the nontrivial design of  $\mathbf{K}$  given the routing mode. First, we obtain the following proposition stating a sufficient condition for system stability. The proof is very simple and a very similar one can be found on Page 30 of [28].

*Proposition 1:* If there exist a matrix  $\mathbf{K}$  with the nonzero element pattern corresponding to a given routing mode, and a positive definite matrix  $\mathbf{P}$ , such that the following LMI holds<sup>7</sup>:

$$\mathbf{A}^T \mathbf{P} + (\mathbf{BKC})^T \mathbf{P} + \mathbf{PA} + \mathbf{PBKC} < 0, \quad (8)$$

then the system is stable for the given routing mode.

Prop. 1 provides a sufficient condition for judging whether a given routing mode can stabilize the system. The difficulty is how to find the suitable matrices  $\mathbf{P}$  and  $\mathbf{K}$ . We take an approach similar to that in [28] via the following two steps:

- 1) Computation of  $\mathbf{P}$ : Assume that  $\mathbf{A}$  is unstable<sup>8</sup>. Suppose that there exists a  $\beta > 0$  such that  $\mathbf{A} - \beta\mathbf{I}$  is unstable and there exists a positive definite matrix  $\mathbf{P}$  such that

$$(\mathbf{A} - \beta\mathbf{I})\mathbf{P} + \mathbf{P}(\mathbf{A} - \beta\mathbf{I})^T = -\mathbf{I}. \quad (9)$$

Note that the rationale of (9) is to make the feasible set of the optimization problem (10) below nonempty.

- 2) Computation of  $\mathbf{K}$ : Given  $\mathbf{P}$ , we obtain  $\mathbf{K}$  by considering the following optimization problem,

$$\begin{aligned} & \max_{\mathbf{K}} \gamma \\ \text{s.t. } & \mathbf{A}^T \mathbf{P} + (\mathbf{BKC})^T \mathbf{P} + \mathbf{PA} + \mathbf{PBKC} + \gamma\mathbf{I} < 0 \\ & K_{ij} = 0, \text{ if sensor } j \text{ is not connected to controller } i \\ & \|\mathbf{K}\|_2 \leq c_K, \end{aligned} \quad (10)$$

where the last constraint is to prevent the feedback matrix  $\mathbf{K}$  from being prohibitively large ( $c_K$  is an upper bound of the 2-norm of  $\mathbf{K}$ ). Using the Schur complement formula [3], it is easy to verify that the constraint  $\|\mathbf{K}\|_2 \leq c_K$  is equivalent to the following linear matrix inequality (Page 33, [28]), which is easier to manipulate mathematically:

$$\begin{pmatrix} -c_K \mathbf{I} & \mathbf{K}^T \\ \mathbf{K} & -\mathbf{I} \end{pmatrix} < 0. \quad (11)$$

It is easy to verify that, if the optimal value of (10) is positive, then the condition (8) holds. Note that the optimization problem in (10) can be readily solved using the theory of LMIs.

Based on the above discussion, we are able to check the stability of the system given the routing mode; meanwhile, we can also construct the stabilizing feedback gain matrix  $\mathbf{K}$  (if possible), as a byproduct. Now, the challenge is how to find a routing mode such that the optimal  $\gamma$  is positive (thus stabilizing the system). Formulated as an optimization problem, the search for a routing mode stabilizing the system dynamics is to maximize  $\gamma$ ; i.e.

$$\begin{aligned} & \max_{\mathcal{R}} \gamma(\mathcal{R}) \\ \text{s.t. } & \mathcal{R} \text{ satisfies the bandwidth constraint,} \end{aligned} \quad (12)$$

<sup>7</sup>For a symmetric matrix  $\mathbf{X}$ ,  $\mathbf{X} < 0$  means that  $\mathbf{X}$  is negative definite.

<sup>8</sup>Otherwise, there is no need to control; the system can converge to zero by itself.

where  $\mathcal{R}$  stands for a routing mode and  $\gamma$  is a function of  $\mathcal{R}$  can be obtained in (10). The difficulty is that it is too complicated to describe the relationship between the routing mode and the objective function  $\gamma$  analytically. Currently, we still do not have any analytical approach to find the stabilizing routing mode. Moreover, it is prohibitively complicated to carry out an exhaustive search. In the next subsection, we will propose a heuristic algorithm to search for such a routing mode in a greedy manner.

2) *Multiple Routing Modes:* It is possible that a single routing mode may not stabilize the cyber physical system or may not be rigorously shown to stabilize the system dynamics. In this case, we need to consider multiple routing modes and let the communication network switch among these modes. For simplicity, we assume that the feedback gain matrix  $\mathbf{K}_q$  corresponding to routing mode  $q$  is obtained from (10). It may result in a better performance if the matrices  $\{\mathbf{K}_q\}_{q=1,\dots,Q}$  are optimized jointly. However, the optimization will be much more complicated and thus will not be studied in this paper. The separated design of  $\mathbf{K}_q$  is also reasonable since a larger  $\gamma$  means more stability; even if the system is unstable, a larger  $\gamma < 0$  implies slower divergence of the system state. Hence, we adopt the matrices obtained in the single routing mode case.

The following proposition provides a sufficient condition for system stability when there are multiple routing modes. Note that the proof is similar to that of Theorem 1 in [7]. Hence, the detailed proof is omitted due to limited space.

*Proposition 2:* Suppose that the feedback gain matrix  $\mathbf{K}_q$  and the mode sequence  $1, 2, \dots, Q, 1, 2, \dots$  have been fixed. If there exist  $Q$  positive definite matrices  $\mathbf{P}_1, \dots, \mathbf{P}_Q$  such that (recall that  $\tau$  is the time interval between two switches of routing modes)

$$\left(e^{\tau \tilde{\mathbf{A}}_q}\right)^T \mathbf{P}_q e^{\tau \tilde{\mathbf{A}}_q} - \mathbf{P}_{q-1} < 0, \forall q = 2, \dots, Q, \quad (13)$$

and

$$\left(e^{\tau \tilde{\mathbf{A}}_1}\right)^T \mathbf{P}_1 e^{\tau \tilde{\mathbf{A}}_1} - \mathbf{P}_Q < 0, \quad (14)$$

then the system is stable.

Since  $\{\tilde{\mathbf{A}}_q\}_{q=1,\dots,Q}$  have been fixed, the LMIs in (13) and (14) can be easily verified and can be rewritten as the following optimization problem:

$$\begin{aligned} & \max_{\mathbf{P}_q, q=1,\dots,Q} \sum_{q=1}^Q \gamma_q \\ \text{s.t. } & \left(e^{\tau \tilde{\mathbf{A}}_q}\right)^T \mathbf{P}_q e^{\tau \tilde{\mathbf{A}}_q} - \mathbf{P}_{q-1} + \gamma_{q-1} < 0, \forall q = 2, \dots, Q, \\ & \left(e^{\tau \tilde{\mathbf{A}}_1}\right)^T \mathbf{P}_1 e^{\tau \tilde{\mathbf{A}}_1} - \mathbf{P}_Q + \gamma_Q < 0. \end{aligned} \quad (15)$$

The rationale for this formulation of the optimization problem is as follows. When we maximize the sum of  $\gamma_q$ , we are driving the variables  $\gamma_q$  to be positive. Once all  $\gamma_q$  become positive, the sufficient conditions in Prop. 2 are satisfied.

Then, the problem of selecting the routing modes is formulated as the following optimization problem:

$$\begin{aligned} & \max_{\{\mathcal{R}_q\}_{q=1,\dots,Q}} \sum_{q=1}^Q \gamma_q \\ \text{s.t. } & \mathcal{R}_q \text{ is feasible, } q = 1, \dots, Q, \end{aligned} \quad (16)$$

where  $\mathcal{R}_q$  is the  $q$ -th routing mode. The rationale is to choose the routing modes to maximize the objective functions in (15), thus trying to make the conditions in Prop. 2 valid.

### C. Heuristic Solution

As we have explained, it is difficult to solve the optimization problems in (12) and (15) analytically. Hence, we propose heuristic algorithms for these problems.

1) *Single Routing Mode*: For the single routing mode case, we search for the connections between the sensors and controllers in a greedy manner. The basic strategy is, in each step, we add a new connection such that the objective function  $\gamma$  is maximized for the existing connections. For the new connection, we choose the closest path from a node having the information of the corresponding sensor to the corresponding controller. For example, in Fig. 1, if there has existed a connection between S1 and G2 via R2, we can simply choose R2 as a relay when looking for the route for the connection between S1 and G3. The reason for the greediness is that the objective function is increased as much as possible at each step, under the constraints on the total communication resources. The details of the proposed heuristic algorithm are provided in Procedure 1.

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#### Procedure 1 Procedure of Finding the Single Routing Mode

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- 1: Initialize the initial routing mode  $\mathcal{R}$  as an empty set.
  - 2: Initialize the unsearched connection set  $\mathcal{U}$  as all possible sensor-controller combinations
  - 3: **while**  $\mathcal{U}$  is nonempty **do**
  - 4:   **for** All connections in  $\mathcal{U}$  **do**
  - 5:     Check the feasibility of the connection subject to the existing connections and the bandwidth constraint.
  - 6:     **if** Feasible **then**
  - 7:       Add the connection to the existing connections to obtain the temporary routing mode.
  - 8:       Solve the matrix equation in (9).
  - 9:       Solve the optimization problem in (10) using the LMI tool and obtain the maximal  $\gamma$ .
  - 10:     **else**
  - 11:       Remove the connection from  $\mathcal{U}$ .
  - 12:     **end if**
  - 13:   **end for**
  - 14:   Choose the connection with the largest  $\gamma$  and obtain the corresponding  $\mathbf{K}$ .
  - 15:   Find the shortest path from the nodes having the information of the sensor to the controller.
  - 16:   Add the connection and the routing information to  $\mathcal{R}$ .
  - 17:   Remove the connection from  $\mathcal{U}$ .
  - 18: **end while**
- 

Although the heuristic algorithm in Procedure 1 may be sub-optimal, we can prove that the heuristic algorithm is optimal for a slightly modified optimization problem and a decoupled cyber physical system with a base station. For this case we can prove the optimality of the greedy algorithm; hence it

justifies the greedy algorithm proposed in Procedure 1. To avoid sidetracking from the development of the main theme, we put the proof of this in Appendix A.

2) *Multiple Routing Modes*: Again, it is very difficult to solve the optimization problem in (16). We propose a heuristic approach that is similar to Procedure 1 for the single mode case. The procedure is also greedy. We use Procedure 1 to obtain  $\mathcal{R}_1$ . Then, we choose  $\mathcal{R}_2$  to maximize  $\gamma_1 + \gamma_2$ , given  $\mathcal{R}_1$ . The procedure for selecting  $\mathcal{R}_2$  is also very similar to Procedure 1. The only difference is that the objective function is changed to  $\gamma_1 + \gamma_2$ . We repeat this procedure until all  $Q$  routing modes have been found. Essentially, we manage to decrease the Lyapunov function at each step. The details are given in Procedure 2.

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#### Procedure 2 Procedure of Finding the Multiple Routing Modes

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- 1: Apply Procedure 1 to obtain  $\mathcal{R}_1$  and  $\mathbf{K}_1$ . Set  $Q = 2$ .
  - 2: If the metric  $\gamma$  in Procedure 1 is positive, stop.
  - 3: **while** The conditions in Prop. 1 are not satisfied for existing modes **do**
  - 4:   For possible modes that are different from  $\mathcal{R}_1$ , apply Procedure 1 to find  $\mathcal{R}_Q$  and  $\mathbf{K}_Q$ .
  - 5:   Using the theory of linear matrix inequalities for the optimization problem in (16);
  - 6:   **if** The metric  $\sum_{q=1}^Q \gamma_q$  is increased **then**
  - 7:     Add  $\mathcal{R}_Q$  to the mode set.
  - 8:     Increase  $Q$  by 1.
  - 9:   **end if**
  - 10:   **if** The number of iterations is more than a threshold **then**
  - 11:     Stop the iteration and claim failure.
  - 12:   **end if**
  - 13: **end while**
  - 14: **if** The conditions in Prop. 1 are satisfied for existing routing modes **then**
  - 15:   Output the routing modes, as well as the feedback matrices  $\{\mathbf{K}_q\}_{q=1,\dots,Q}$ .
  - 16: **end if**
- 

Note that the proposed routing algorithm has addressed the two challenges discussed in the introduction, implicitly or explicitly, within the framework of optimization:

- Uncertain destinations: In the optimization, each sensor can find the destination controllers that mostly contribute to the objective function in the optimization problem, within the constraints on the communication resources.
- Multiple routing modes: The algorithm can find multiple routing modes that stabilize the system dynamics, if a single routing mode cannot.

## IV. MULTICAST ROUTING WITH SMALL DELAYS

In the previous section, we ignored the delay in order to simplify the analysis. However, in practice, there always exists communication delay. Hence, based on the discussion of the no delay case, we will study the multicast routing algorithm when the delay is non-negligible. We will first write down the dynamics when delay exists. Then, we formulate the routing problem as an optimization problem and finally propose a heuristic algorithm similar to Procedure 1. Note that we consider only the single routing mode in this section, for simplicity of analysis.

### A. Dynamics with Delay

We fix a routing mode  $\mathcal{R}$ . Then, the system dynamics can be written as

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \sum_{(n,m) \in \mathcal{R}} \mathbf{b}(:,m)\mathbf{k}(n,:) \mathbf{C}\mathbf{x}(t - d_{nm}), \quad (17)$$

where  $(n,m) \in \mathcal{R}$  means that the connection between sensor  $n$  and controller  $m$  is established,  $\mathbf{b}(:,m)$  is the  $m$ -th column of matrix  $\mathbf{B}$ ,  $\mathbf{k}(n,:)$  is the  $n$ -th row of matrix  $\mathbf{K}$ , and  $d_{nm}$  is the delay between sensor  $n$  and controller  $m$ .

The stability of systems with delayed feedback has been widely studied using the Lyapunov-Krosovskii functional [22]. However, the condition of stability is expressed using multiple LMIs with multiple undetermined matrices, which is very difficult to analyze. In this paper, we consider the special case in which the delay is small but non-negligible. Such an assumption is reasonable due to the fast speed of modern wireless networks compared with the time scales of dynamics in many typical cyber physical systems (e.g., the time scale for wide area situational awareness in smart grid is on the order of tens of milliseconds). Hence, on assuming  $d_{nm}$  is small, we can expand  $\mathbf{x}(t - d_{nm})$  as

$$\begin{aligned} & \mathbf{x}(t - d_{nm}) \\ &= \mathbf{x}(t) - d_{nm}\dot{\mathbf{x}}(t) + o(d_{nm}) \\ &\approx \mathbf{x}(t) - d_{nm} \left( \mathbf{A}\mathbf{x}(t) + \sum_{(n,m) \in \mathcal{R}} \mathbf{b}(:,m)\mathbf{k}(n,:) \mathbf{C}\mathbf{x}(t) \right) \\ &= \left( \mathbf{I} - d_{nm} \left( \mathbf{A} + \sum_{(n,m) \in \mathcal{R}} \mathbf{b}(:,m)\mathbf{k}(n,:) \mathbf{C} \right) \right) \mathbf{x}(t). \end{aligned} \quad (18)$$

Then, the system dynamics can be approximated by

$$\dot{\mathbf{x}}(t) \approx \tilde{\mathbf{A}}\mathbf{x}(t), \quad (19)$$

where

$$\tilde{\mathbf{A}} \triangleq (\mathbf{A} + \mathbf{B}\mathbf{K}\mathbf{C})(\mathbf{I} - \mathbf{B}\mathbf{D}\mathbf{K}\mathbf{C}), \quad (20)$$

where  $\mathbf{D}$  is the matrix containing the delays, namely

$$D_{nm} = \begin{cases} d_{nm}, & \text{if } n \text{ and } m \text{ are connected} \\ 0, & \text{otherwise} \end{cases}. \quad (21)$$

The following proposition shows that the stability of the system can be assured by the Lyapunov inequality given sufficiently small time delays. The proof is given in Appendix B.

*Proposition 3:* For sufficiently small time delays, the system in (19) is stable if there exists a positive definite matrix  $\mathbf{P}$  such that the following inequality holds:

$$\mathbf{P}^T \tilde{\mathbf{A}} + \tilde{\mathbf{A}}^T \mathbf{P} < 0. \quad (22)$$

### B. Optimization Problem Formulation

Again, we formulate the routing problem as an optimization problem in a manner similar to that in the delay-free case. However, in contrast to the delay-free case, we observe that  $\tilde{\mathbf{A}}$  is nonlinear in  $\mathbf{K}$ , which prevents a straightforward application of LMIs. One effective approach is to consider the nonlinear term, whose scale is approximately proportional to the delays, as a perturbation to the system dynamics. Hence, we rewrite the system dynamics as

$$\dot{\mathbf{x}} = (\mathbf{A} + \mathbf{B}\mathbf{K}\mathbf{C})\mathbf{x} + \mathbf{h}(\mathbf{x}), \quad (23)$$

where the higher order terms are discarded and

$$\mathbf{h}(\mathbf{x}) = -\mathbf{B}\mathbf{D}\mathbf{K}(\mathbf{A} + \mathbf{B}\mathbf{K}\mathbf{C})\mathbf{x}. \quad (24)$$

It is easy to verify that

$$\mathbf{h}^T(\mathbf{x})\mathbf{h}(\mathbf{x}) \leq \alpha^2 \mathbf{x}^T \mathbf{x}, \quad (25)$$

where

$$\alpha = \max_{mn} \{d_{mn}\} c_K \|\mathbf{B}\|_2 (\|\mathbf{A}\|_2 + \|\mathbf{B}\|_2 c_K). \quad (26)$$

According to the argument in [28] (Page 32), if there exists a  $\gamma < 0$  such that

$$\begin{pmatrix} \mathbf{A}_K^T \mathbf{P} + \mathbf{P} \mathbf{A}_K + \gamma \alpha \mathbf{I} & \mathbf{P} \\ \mathbf{P} & \gamma \mathbf{I} \end{pmatrix} < 0, \quad (27)$$

where  $\mathbf{A}_K = \mathbf{A} + \mathbf{B}\mathbf{K}\mathbf{C}$ , then the Lyapunov Inequality constraint  $\mathbf{P}^T \tilde{\mathbf{A}} + \tilde{\mathbf{A}}^T \mathbf{P} < 0$  holds. Hence, we can formulate the optimization problem as

$$\begin{aligned} & \max_{\mathcal{R}, \{\mathbf{K}\}, \gamma} \gamma \\ \text{s.t.} \quad & \text{LMI (27) holds} \\ & \|\mathbf{K}\|_2 \leq c_K. \\ & \gamma < 0. \end{aligned} \quad (28)$$

Note that many details of the routing delay have been omitted in the proposed optimization problem; only the maximum delay is taken into account. This will cause performance loss. However, numerical results show that the proposed optimization problem formulation can still find stabilizing routes.

To solve the optimization problem, we follow the same algorithm of Procedure 1 to find the stabilizing routing scheme, namely choosing the paths minimizing the objective function in a greedy manner. Hence, the details of the algorithm are omitted due to space limitations.

## V. APPLICATION IN DEG VOLTAGE CONTROL OF SMART GRID

Recent years have witnessed the rapid growth of DEGs due to the pressing demand for reliability and security of the electricity power grid. DEGs are coupled into the main power network at the points of common coupling (PCCs). An important task of DEGs is to control the voltages at the PCCs or remote locations, which can be accomplished by power electronics (PE) interfaces. Each PE interface consists of an inverter and a DC-side capacitor. There is a coupling inductor between the inverter and the rest of the system. Note that the voltage control of a DEG is different from that of traditional generators in large power grids. The DEG voltage control is

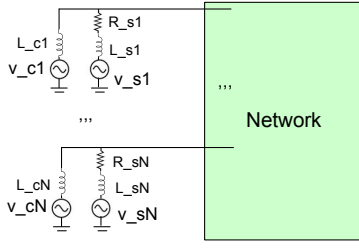


Fig. 2. An illustration of the model of multiple DEGs connecting to the power network.

based on PE interfaces, whose dynamics are much faster, while the voltage control in traditional power grids is based on the allocation of reactive power which has a much slower time scale.

In this paper, we adopt the model of a power network from [17], as shown in Fig. 2. We assume that there are  $N$  DEGs, modeled as voltage sources whose voltages are denoted by  $\{v_c^n(t)\}_{n=1,\dots,N}$ . A coupling inductor exists between each DEG and the rest of the network. There could be a voltage source (substation) at each bus, whose voltages are denoted by  $\{v_s^n(t)\}_{n=1,\dots,N}$ . When there is no source at bus  $n$ , we can set  $v_s^n = 0$ . To simplify the analysis, we assume  $v_s^n = 0$  for all  $n = 1, \dots, N$ ; i.e., only DEGs are connected to the power network. Then, the nodal voltage equations are given by (in the domain of Laplace transformation)

$$\mathbf{Y}(s) \begin{pmatrix} v_{t1}(s) \\ \vdots \\ v_{tN}(s) \end{pmatrix} = \begin{pmatrix} \frac{v_{c1}(s)}{sL_{c1}} \\ \vdots \\ \frac{v_{cN}(s)}{sL_{cN}} \end{pmatrix}, \quad (28)$$

where  $\mathbf{Y}$  is the admittance matrix of the power network,  $\{v_{tn}(s)\}_{n=1,\dots,N}$  are the voltages at the PCCs and  $\{L_{cn}\}_{n=1,\dots,N}$  are the coupling inductors. Defining  $\mathbf{v}_t = (v_{t1}, \dots, v_{tN})^T$ ,  $\mathbf{v}_c = (v_{c1}, \dots, v_{cN})^T$  and  $\mathbf{L} = \text{diag}(L_{c1}, \dots, L_{cN})$ , we can rewrite the voltage equation in (28) as

$$\mathbf{Y}(s)\mathbf{v}_t(s) = \frac{1}{s}\mathbf{L}\mathbf{v}_c(s), \quad (29)$$

which fully describes the dynamics of the voltage in the power grid.

Then, we can convert the Laplace transform to the time domain, thus obtaining the linear dynamics in (1). We assume that the purpose of the voltage control is to keep the voltages at PCCs to be a reference value  $V_{ref}$ . We also assume that, when the system reaches equilibrium,  $\mathbf{v}_t = V_{ref}$  given a proper  $\mathbf{v}_c$ . Hence, we define the system state as the deviation of the voltages from the reference value; i.e.,  $\mathbf{x} = \mathbf{v}_f - V_{ref}$ . Thus, the dynamics of the system state can be written in the form of (1). We will discuss the details of the conversion for a 4-bus example in the numerical simulations.

## VI. NUMERICAL SIMULATION

In this section, we study a simple 4-bus example and demonstrate the performance of the algorithms proposed in this paper.

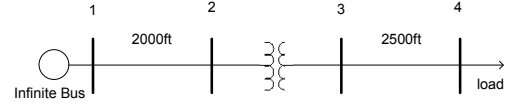


Fig. 3. An illustration of the four bus model.

### A. Simulation Configuration

1) *Power Grid*: For simplicity, we consider a simple example, a 4-bus model of the distribution test feeders proposed by the IEEE distribution test feeder working group, which is illustrated in Fig. 3. The details can be found in [10]. The admittance matrix is given in (30) in the top of next page.

We obtain the system matrices, which are given by

$$\mathbf{A} = \begin{pmatrix} 0.1759 & 0.1768 & 0.5110 & 1.0360 \\ -0.3500 & -0.0000 & -0.0000 & -0.0000 \\ -0.5442 & -0.4748 & -0.4088 & -0.8288 \\ -0.1197 & -0.5546 & -0.9688 & -1.0775 \end{pmatrix} \times 10^3, \quad (31)$$

and

$$\mathbf{B} = \begin{pmatrix} 0.0008 & 0.3342 & 0.5251 & -1.0360 \\ -0.3500 & -0.0000 & -0.0000 & -0.0000 \\ -0.0693 & -0.0661 & -0.4201 & -0.8288 \\ -0.4349 & -0.4142 & -0.1087 & -1.0775 \end{pmatrix} \times 10^3. \quad (32)$$

Note that both matrices are obtained from the Laplace transform in (28). The details are omitted due to space limitations. It is easy to verify that not all eigenvalues of  $\mathbf{A}$  are negative (or have a negative real part). Hence, the system is unstable when there is no feedback control.

2) *Communication Network*: We assume that there are two arrays of relay nodes, each containing 4 nodes, between the sensors and controllers, thus forming a  $4 \times 4$  array in the plane, as illustrated in the upper part of Fig. 4. Hence, each packet from a sensor must pass three hops to reach a controller. For simplicity, we assume that each node can forward the packet of only one sensor, i.e.,  $w_{ab} = 1$ ; the transmission of one packet can reach all next-hop neighbors. We also assume that each controller can receive packets from multiple relay nodes simultaneously.

### B. Numerical Results

Based on the above configuration of the power network and communication network, we carry out simulations for multiple situations. Throughout all simulations, we fix  $c_K = 5$ ; i.e., the 2-norm of the feedback gain matrix  $\mathbf{K}$  cannot be larger than 5. Note that we used the robust control toolbox of Matlab to solve the optimization problems with LMI constraints.

1) *Single Mode Case - No Delay*: We first tested the performance of Procedure 1 when we consider only one routing mode. The network topology is given in the upper part of Fig. 4. The routing scheme obtained from Procedure 1 is given in the lower part of Fig. 4. We observe that it is not necessary for each controller to receive packets from all sensors. For example, controller 4 does not receive any reports from sensors 1 and 2. Figure 5 shows the evolution of the metric  $\gamma$  and the 2-norm of  $\mathbf{K}$  as more and more paths are selected. We observe that the metric increases monotonically while  $\|\mathbf{K}\|_2$  is constrained within  $c_K = 5$ . Fig. 6 shows the maximum eigenvalue of the dynamics with respect to the rounds of routing. We observe that the system becomes stable



$$\mathbf{Y}(s) = \begin{pmatrix} \frac{1}{0.1750+0.0005s} & -\frac{1}{0.1750+0.0005s} & 0 & 0 \\ -\frac{1}{0.1750+0.0005s} & \frac{1}{0.1750+0.0005s} + \frac{1}{0.1667+0.0004s} & -\frac{1}{0.1667+0.0004s} & 0 \\ 0 & -\frac{1}{0.1667+0.0004s} & \frac{1}{0.1667+0.0004s} + \frac{1}{0.2187+0.0006s} & 0 \\ 0 & 0 & -\frac{1}{0.2187+0.0006s} & \frac{1}{0.2187+0.0006s} + \frac{1}{12.3413+0.0148s} \end{pmatrix} + \frac{1}{Ls} \mathbf{I}. \quad (30)$$

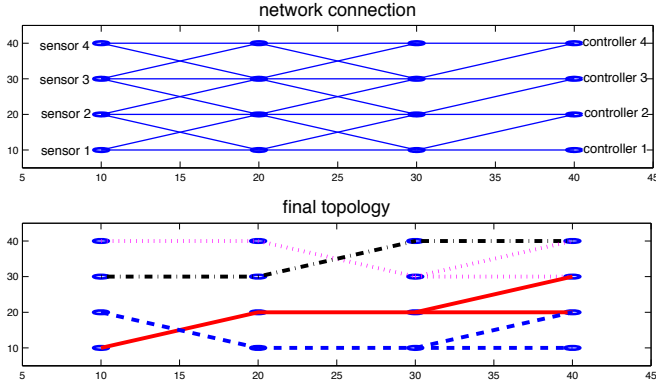


Fig. 4. Network topology obtained from Procedure 1 in the single-mode and delay-free case.

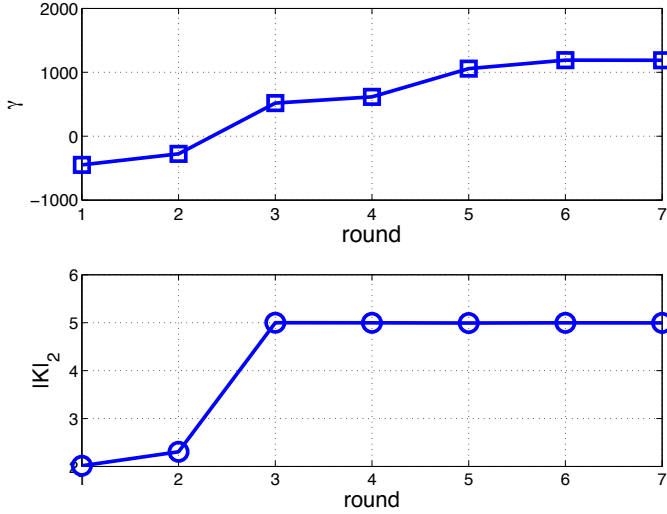


Fig. 5. The evolution of  $\gamma$  and  $\|\mathbf{K}\|_2$  in the single-mode and delay-free case.

(the maximum eigenvalue becomes negative) after several rounds of routing.

2) *Multiple Mode Case - No Delay*: From the network topology in Fig. 4, which can be stabilized by a single routing mode, we remove some links and result in the topology shown in Fig. 7. The routing scheme obtained from Procedure 1 is also shown in Fig. 7. However, from Fig. 8 which shows the evolution of  $\gamma$  and  $\|\mathbf{K}\|_2$ , we observe that the routing scheme does not result in a stabilizing feedback control since  $\gamma$  is always negative. Note that this does not mean that the topology in Fig. 7 cannot be stabilized by a single routing mode since Procedure 1 does not guarantee an optimal routing scheme; however, at least our current routing algorithm is unable to find a single stabilizing routing scheme for the network topology. We also applied Procedure 2 to seek for multiple routing

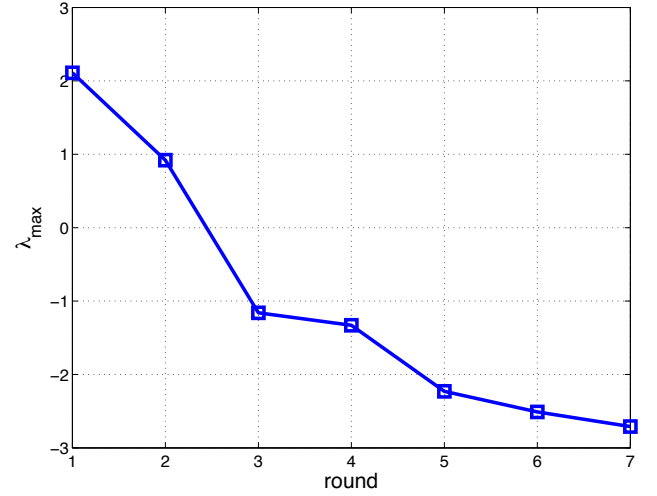


Fig. 6. The maximum eigenvalue of the linear dynamics.

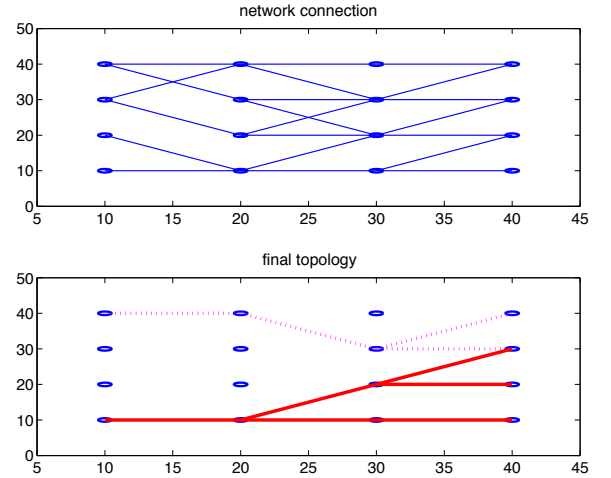


Fig. 7. Network topology obtained from Procedure 1 in the single-mode and delay-free case: routing failure.

modes. The result is shown in Fig. 9, in which  $Q = 2$  (i.e., the system can be stabilized by two consecutive routing modes). We observe that, in the first mode, reports from sensor 4 are sent to controllers 3 and 4; in the second mode, reports from sensor 3 are delivered to controller 3 and 4; in both cases, reports from sensor 1 are multicast to sensors 1, 2 and 3. Hence, if it is difficult to find a single routing mode to stabilize the system, we can always try to find multiple routing modes and apply them alternatively.

We can also use the analysis to find critical links, which are defined as the links whose removal will cause the instability of the system. For the topology shown in Fig. 10, we found



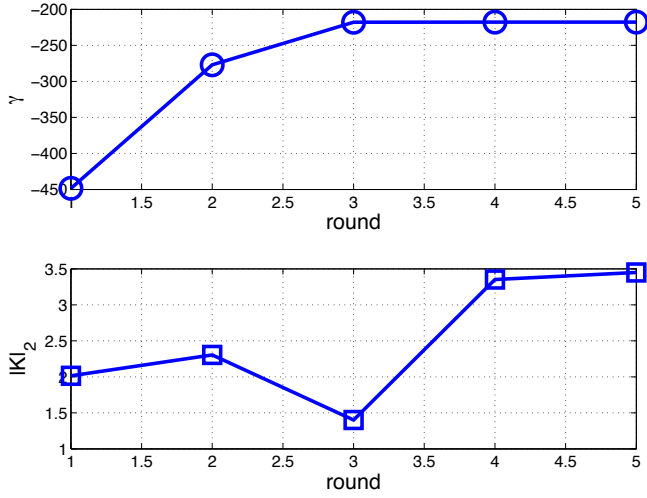
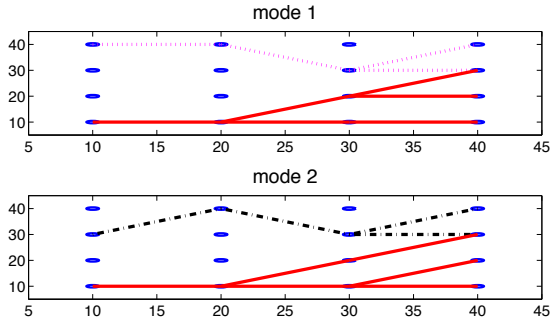
Fig. 8. The evolution of  $\gamma$  and  $\|K\|_2$  in the single-mode and delay-free

Fig. 9. Network topology obtained from Procedure 2 in the multiple-mode and delay-free case.

that the only critical link is the link from sensor 1 to its neighbor. A reasonable explanation is given as follows. We notice that this link is the only way to convey the reports from sensor 1. Without this link, observations at sensor 1 will be invisible. From the expression of  $\mathbf{A}$  and  $\mathbf{B}$ , we observe that both the state at sensor 1 and the action of controller 1 have strong impacts on the state at sensor 2. Hence, without the observation at sensor 1, it is difficult for controller 1 to make its decision, thus making the state at sensor 2 unstable. The importance of the observations at sensor 1 can also be observed in Fig. 9 since, in both modes, the reports of sensor 1 are visible.

We also tested the performance of the proposed routing algorithm in the case of non-negligible delays. We assume that the delay between two nodes is uniformly distributed between 0.5ms and 1ms. For one realization of the delays, the obtained network topology is given in Fig. 11, which is shown by simulations to stabilize the dynamics.

## VII. CONCLUSIONS

In this paper, we have studied multicast routing for decentralized control in cyber physical systems and applied the proposed algorithms to distributed voltage control in smart grid. To address the challenges of uncertain destinations and multiple routing modes, we have formulated the routing problem as a series of optimization problems, using the theories

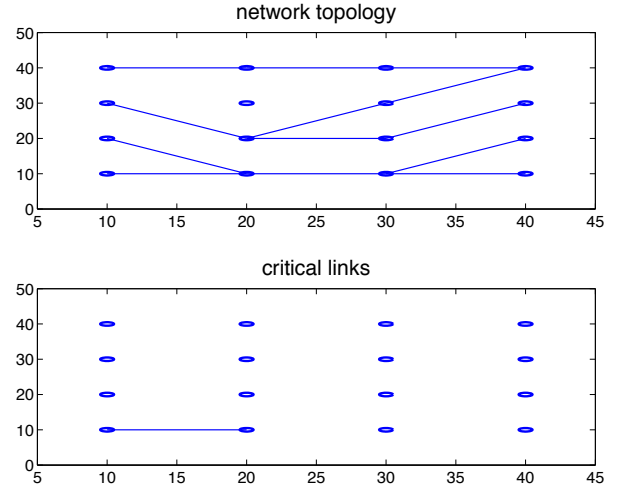


Fig. 10. Example with one critical link.

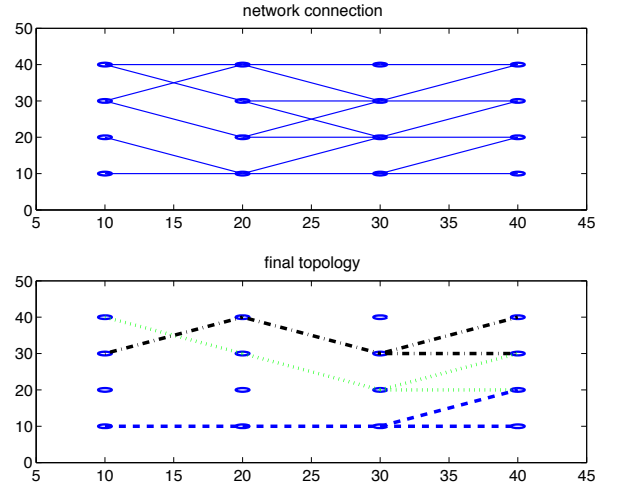


Fig. 11. Network topology when the delay is non-negligible.

of LMIs and hybrid systems. We have considered both cases of no communication delay and small communication delay. For both cases, we have proposed heuristic greedy algorithms to solve the optimization problems. As a case study, we have considered voltage control in a 4-bus power grid having a communication network. Numerical simulation results have shown that our proposed algorithms can find stabilizing routes for the power grid.

## APPENDIX A JUSTIFICATION OF GREEDY ALGORITHM

*Proposition 4:* Consider a decoupled system in which the matrices  $\mathbf{A}$ ,  $\mathbf{B}$  and  $\mathbf{C}$  are all diagonal. Assume that there is only one relay node, which can be considered as a base station, as illustrated in Fig. 12. The sensors and controllers cannot talk to each other directly. Each connection must route through the base station. The base station can support at most  $Q$  connections. Then, the algorithm in Procedure 1 is optimal for the following optimization problem:

$$\begin{aligned}
& \max_{\mathbf{K}, \mathcal{R}, \mathbf{A}} \text{trace}(\mathbf{A}) \\
& \text{s.t.} \quad \mathbf{A}^T \mathbf{P} + (\mathbf{BKC})^T \mathbf{P} + \mathbf{PA} + \mathbf{PBKC} + \mathbf{A} < 0 \\
& \quad K_{ij} = 0, \text{ if sensor } j \text{ not connected to controller } i \\
& \quad \mathcal{R} \text{ satisfies the bandwidth constraint} \\
& \quad \|\mathbf{K}\|_2 \leq c_K,
\end{aligned} \tag{33}$$

where  $\mathbf{P}$  is determined by (9).

*Proof:* The proof is based on the theory of matroids [13]. First, it is easy to verify that  $\mathbf{P}$  is diagonal, due to equation (9) and the assumption that  $\mathbf{A}$  is diagonal. Since the system dynamics are decoupled, the control is also decoupled, i.e.,  $\mathbf{K}$  is diagonal. Hence,  $\mathbf{A}$  is also diagonal.

Next, we will show that the problem has a matroid structure [13]. A matroid is defined as a structure  $(\mathcal{E}, \mathcal{M})$  in which  $\mathcal{E}$  is a finite set of elements and  $\mathcal{M}$  is a family of subsets of  $\mathcal{E}$ . The structure  $(\mathcal{E}, \mathcal{M})$  satisfies the following conditions:

- The empty set belongs to  $\mathcal{M}$ . For any  $I \in \mathcal{M}$ , all subsets of  $I$  belong to  $\mathcal{M}$ .
- If  $I_p$  and  $I_{p+1}$  are both elements in  $\mathcal{M}$ , which have  $p$  and  $p+1$  elements, respectively, then there exists an  $e \in I_{p+1} - I_p$  such that  $I_p + e \in \mathcal{M}$ .

For the optimization problem in (33), we define the elements in  $\mathcal{E}$  as the sensor-controller connections and  $\mathcal{M}$  as the sets of connections satisfying the bandwidth constraints. Then, we need to verify the two conditions for a matroid:

- An empty set in  $\mathcal{M}$  means no communications, which surely satisfies the communication constraint. A subset of feasible connections also satisfies the constraint. Hence, the first condition is verified.
- Consider  $I_p, I_{p+1} \in \mathcal{M}$ . Obviously, there are  $p$  and  $p+1$  connections supported by the base station. Hence, any connection in  $I_{p+1} - I_p$  can be added to  $I_p$  and still satisfy the communication constraint. This verifies the second condition.

When a connection, say connecting sensor  $i$  and controller  $i$ , is chosen, we obtain a positive value given by

$$\Lambda_{ii} = \max_{|x| < c_K} 2A_{ii}P_{ii} + 2xB_{ii}C_{ii}P_{ii}. \tag{34}$$

When the connection of sensor  $j$  and controller  $j$  is not selected, the corresponding  $\Lambda_{jj} = 0$ . Hence, we can assign each connection in  $\mathcal{E}$  a weight given by (34). Then, the optimization problem in (33) is equivalent to selecting a set in  $\mathcal{M}$  such that the sum of weights in the set is maximized. It has been shown in [13] that a greedy algorithm, described in [13], can achieve the optimum. It is easy to verify that the algorithm in Procedure 1 is a greedy one of this type. This concludes the proof. ■

*Remark 1:* Note that the optimization in (33) is very similar to a combination of (10) and (12). The only difference is that we replace  $\gamma\mathbf{I}$  with a more generic matrix  $\mathbf{A}$ . Hence, the optimality established in Prop. 4 provides a justification for the algorithm in Procedure 1. The simplified system in Prop. 1 is also reasonable for practical applications with a cellular communication infrastructure when the sensor-control pairs are mutually independent or the coupling is very weak.

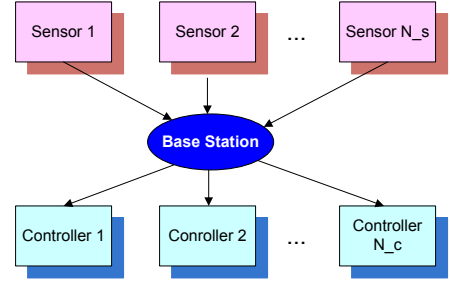


Fig. 12. An illustration of the star topology in Prop. 4.

Note that the optimality in Prop. 4 may be extended to the case of multiple base stations by employing the theory of matroid intersection<sup>9</sup> [13], which is beyond the scope of this paper.

## APPENDIX B PROOF OF PROP. 3

*Proof:* The proof follows the analysis of robust control with small perturbations in [28]. We denote by  $\mathbf{h}(\mathbf{x})$  the residual after the linearization, which is a nonlinear function of  $\mathbf{x}$ ; i.e.,

$$\dot{\mathbf{x}}(t) = \tilde{\mathbf{A}}\mathbf{x}(t) + \mathbf{h}(\mathbf{x}). \tag{35}$$

Suppose that the matrix  $\mathbf{P}$  in (22) exists. We define the Lyapunov function as

$$V(\mathbf{x}) = \mathbf{x}^T \mathbf{P} \mathbf{x}. \tag{36}$$

Then, we have  $V(\mathbf{x}) > 0$ , if  $\mathbf{x} \neq 0$ , since  $\mathbf{P}$  is positive definite. Moreover, we have

$$\begin{aligned}
\dot{V}(\mathbf{x}(t)) &= \mathbf{x}^T \left( \tilde{\mathbf{A}}^T \mathbf{P} + \mathbf{P}^T \tilde{\mathbf{A}} \right) \mathbf{x} \\
&+ \mathbf{x}^T \mathbf{P} \mathbf{h}(\mathbf{x}) + \mathbf{h}(\mathbf{x})^T \mathbf{P} \mathbf{x}.
\end{aligned} \tag{37}$$

Due to the Lyapunov inequality in (22), we have

$$\mathbf{P}^T \tilde{\mathbf{A}} + \tilde{\mathbf{A}}^T \mathbf{P} < -\delta \mathbf{I}, \tag{38}$$

for a certain  $\delta > 0$ . Then, the first two terms in (37) satisfy

$$\mathbf{x}^T (\mathbf{A}^T \mathbf{P} + \mathbf{P}^T \mathbf{A}) \mathbf{x} < -\delta \|\mathbf{x}\|^2. \tag{39}$$

When the delays are sufficiently small, we have  $\|\mathbf{h}(\mathbf{x})\| < \epsilon \|\mathbf{x}\|$  where  $\epsilon > 0$  is also correspondingly small. Then, the last two terms in (22) satisfy

$$\mathbf{x}^T \mathbf{P} \mathbf{h}(\mathbf{x}) + \mathbf{h}(\mathbf{x})^T \mathbf{P} \mathbf{x} < \epsilon \lambda_{\mathbf{P}}^{\max} \|\mathbf{x}\|^2, \tag{40}$$

where  $\lambda_{\mathbf{P}}^{\max}$  is the largest eigenvalue of  $\mathbf{P}$ . Hence, we have

$$\dot{V}(\mathbf{x}(t)) < (-\delta + \epsilon \lambda_{\mathbf{P}}^{\max}) \|\mathbf{x}\|^2, \tag{41}$$

which is negative for sufficiently small  $\epsilon$ . Hence, the system is stable. ■

<sup>9</sup>Each base station with the corresponding communication constraint can be considered as a matroid. Then, the connections satisfying the constraints of all base stations can be considered as a matroid intersection.

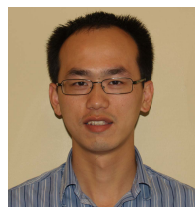
## REFERENCES

- [1] V. Ajarapu, *Computational Techniques for Voltage Stability Assessment and Control*, Springer, New York, 2006.
- [2] J. Bai, E. P. Eysi, Y. Xue and X. D. Koutsoukos, "Dynamic tuning retransmission limit of IEEE 802.11 MAC protocol for networked control systems," in *Proc. 1st International Workshop on Cyber-Physical Networking Systems (CPNS)*, Shanghai, China, 2011.
- [3] S. Boyd, L. El Ghaoui, E. Feron and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*, Society of Industrial and Applied Mathematics, Philadelphia, PA, 1994.
- [4] F. Bullo, J. Cortés and S. Martinez, *Distributed Control of Robotic Networks: A Mathematical Approach to Motion Coordination Algorithms*, Princeton University Press, Princeton, NJ, 2009.
- [5] L. Chen, X. Liu, Q. Wang and Y. Wang, "A real-time multicast routing scheme for multi-hop switched feedbuses," in *Proc. IEEE Conference on Computer Communications (Infocom)*, Shanghai, China, 2011.
- [6] DOE Smart Grid website, [Online]. Available: <http://www.oe.energy.gov/smartgrid.htm>, accessed in January 2010.
- [7] J. C. Geromel and P. Colaneri, "Stability and stabilization of discrete time switched systems," *International Journal of Control*, vol.79, pp.719–729, July 2006.
- [8] V. Gupta, "On an estimation oriented routing protocol," in *Proc. American Control Conference (ACC)*, Baltimore, MD, 2010.
- [9] J. P. Hespanha, P. Naghshtabrizi and Y. Xu, "A survey of recent results in NCS," *Proc. IEEE*, Vol. 95, pp. 138–162, Jan. 2007.
- [10] IEEE PES Distribution System Analysis Subcommittee, *Distribution Test Feeders*, available at <http://ewh.ieee.org/soc/pes/dsacom/testfeeders/index.html>
- [11] ISO New England Inc., *Overview of the Smart Grid: Policies, Initiatives and Needs*, Feb. 17, 2009.
- [12] V. P. Kompella, J. C. Pasquale and G. C. Polyzos, "Multicast routing for multimedia communication," *IEEE/ACM Trans. Netw.*, vol.1, no.3, pp.286–292, June, 1993.
- [13] E. Lawler, *Combinatorial Optimization: Networks and Matroids*, Dover, New York, 1976.
- [14] C. Langbort and V. Gupta, "Minimal interconnection topology in distributed control design," in *Proc. American Control Conference (ACC)*, Minneapolis, MN, 2006.
- [15] E. Lee, "Cyber physical systems: Design challenges," University of California, Berkeley Technical Report, 2008.
- [16] H. Li and Z. Han, "Distributed scheduling of wireless communications for voltage control in micro smart grid," *IEEE Globecom Workshop on Smart Grid Communications*, Houston, TX, 2011.
- [17] H. Li, F. Li, Y. Xu, D. T. Rzy and J. D. Kueck, "Adaptive voltage control with distributed energy resources: Algorithm, theoretical analysis, simulation, and field test verification," *IEEE Trans. Power Syst.*, vol. 25, no. 3, pp.1638–1647, Aug., 2010.
- [18] X. Liu and A. Goldsmith, "Wireless medium access control in distributed control systems," in *Proc. Annual Allerton Conference on Communications, Control and Computing*, Monticello, IL, 2003.
- [19] X. Liu and A. Goldsmith, "Wireless network design for distributed control," in *Proc. IEEE Conference on Decision and Control (CDC)*, Paradise Island, Bahamas, 2004.
- [20] J. Lunze and F. L. Lararigue, *Handbook of Hybrid Systems Control: Theory, Tools and Applications*, Cambridge Univ. Press, Cambridge, UK, 2009.
- [21] J. Machowski, J. W. Bialek and J. R. Bumby, *Power System Dynamics: Stability and Control*, 2nd edition, Wiley, New York, 2008.
- [22] M. S. Mahmoud, *Switched Time-delay Systems*, Springer, Berlin, 2010.
- [23] P. Marwedel, "Embedded and cyber-physical systems in a nutshell," in *Proc. Design Automation Conference (DAC)*, Anaheim, CA, 2010.
- [24] L. Su, B. Ding, Y. Yang, T. F. Abdelzaher, G. Cao and J. C. Hou, "oCast: Optimal multicast routing protocol for wireless sensor networks," in *Proc. IEEE International Conference on Network Protocols*, Princeton, NJ, 2009.
- [25] M. Wu, Y. He and J. She, *Stability Analysis and Robust Control of Time-Delay Systems*, Springer, New York, 2010.
- [26] L. Xiao, M. Johansson, H. Hindi, S. Boyd and A. Goldsmith, "Joint optimization of communication rates and linear systems," *IEEE Trans. Automat. Contr.*, vol.48, pp.148–153, Jan. 2003.
- [27] X. Yu and K. Tomovic, "Application of linear matrix inequalities for frequency control with communication delays," *IEEE Trans. Power Syst.*, vol.19, pp.1508–1515, Aug. 2004.
- [28] A. I. Zečević and D. D. Šiljak, *Control of Complex Systems: Structural Constraints and Uncertainty*, Springer, Berlin, 2010.



**Husheng Li** (S'00-M'05) received the BS and MS degrees in electronic engineering from Tsinghua University, Beijing, China, in 1998 and 2000, respectively, and the Ph.D. degree in electrical engineering from Princeton University, Princeton, NJ, in 2005.

From 2005 to 2007, he worked as a senior engineer at Qualcomm Inc., San Diego, CA. In 2007, he joined the EECS department of the University of Tennessee, Knoxville, TN, as an assistant professor. His research is mainly focused on wireless communications and smart grid. Dr. Li is the recipient of the Best Paper Award of the *EURASIP Journal of Wireless Communications and Networks*, 2005 (together with his PhD advisor: Prof. H. V. Poor), the best demo award of Globecom 2010 and the Best Paper Award of ICC 2011.



**Lifeng Lai** (M'07) received the B.E. and M.E. degrees from Zhejiang University, Hangzhou, China, in 2001 and 2004 respectively, and the Ph.D. degree from the Ohio State University, Columbus, OH, in 2007. He was a Postdoctoral Research Associate at Princeton University, Princeton, NJ, from 2007 to 2009. He is now an Assistant Professor at the University of Arkansas, Little Rock. His research interests include wireless communications, security, and smart grid.

Dr. Lai was a Distinguished University Fellow of the Ohio State University from 2004 to 2007. He is a co-recipient of the Best Paper Award from IEEE Global Communications Conference (Globecom), 2008, and the Best Paper Award from IEEE Conference on Communications (ICC), 2011. He received the National Science Foundation CAREER Award in 2011.



**H. Vincent Poor** (S72, M77, SM82, F87) received the Ph.D. degree in EECS from Princeton University in 1977. From 1977 until 1990, he was on the faculty of the University of Illinois at Urbana-Champaign. Since 1990 he has been on the faculty at Princeton, where he is the Michael Henry Strater University Professor of Electrical Engineering and Dean of the School of Engineering and Applied Science. Dr. Poor's research interests are in the areas of stochastic analysis, statistical signal processing and information theory, and their applications in wireless

networks and related fields including social networking and smart grid. Among his publications in these areas are the recent books *Classical, Semi-classical and Quantum Noise* (Springer, 2012) and *Smart Grid Communications and Networking* (Cambridge University Press, 2012).

Dr. Poor is a member of the National Academy of Engineering and the National Academy of Sciences, a Fellow of the American Academy of Arts and Sciences, and an International Fellow of the Royal Academy of Engineering (U. K.). He is also a Fellow of the Institute of Mathematical Statistics, the Optical Society of America, and other organizations. In 1990, he served as President of the IEEE Information Theory Society, and in 2004-07 he served as the Editor-in-Chief of the *IEEE Transactions on Information Theory*. He received a Guggenheim Fellowship in 2002 and the IEEE Education Medal in 2005. Recent recognition of his work includes the 2010 IET Ambrose Fleming Medal for Achievement in Communications, the 2011 IEEE Eric E. Sumner Award, the 2011 Society Award of the IEEE Signal Processing Society, and the degree of D.Sc. *honoris causa* from the University of Edinburgh, conferred in June 2011, and from Aalborg University, conferred in April 2012.